On the Mixing of the Scalar Mesons $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$

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Abstract

Based on a 3×3 mass matrix describing the mixing of the scalar states $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$, the hadronic decays of the three states are investigated. Taking into account the two possible assumptions concerning the mass level order of the bare states $|N\rangle = |u\bar{u} + d\bar{d}\rangle/\sqrt{2}$, $|S\rangle = |s\bar{s}\rangle$ and $|G\rangle = |gg\rangle$ in the scalar sector, $M_G > M_S > M_N$ and $M_G > M_N > M_S$, we obtain the glueball-quarkonia content of the three states by solving the unlinear equations. Some predictions about the decays of the three states in two cases are presented, which can provide a stringent consistency check of the two assumptions.

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I. INTRODUCTION

The existence of glueball states made of gluons is one of the important predictions of QCD. Discovery and confirmation of these glueball states would be the strong support to the QCD theory. Therefore, the search for and identifying the glueball states have been a very excited and attractive research subject. The abundance of $q\bar{q}$ mesons and the possible mixing of glueballs and ordinary mesons make the current situation with the identification of the glueball states rather complicated. However, some progress has been made in the glueball sector. By studying the mixing between quarkonia and glueball to study the properties of glueballs or identify the glueball states is an appealing approach [1–5].

In contrast to the vector and tensor mesons, the identification of the scalar mesons is a long standing puzzle. In particular, the $I=0$, $J^{PC}=0^{++}$ sector is the most complex one both experimentally and theoretically. The quark model predicts that there are two mesons with $I=0$ in the ground $^3P_0$ $q\bar{q}$ nonet, but apart from the state $f_J(1710)$ with $J=0$ or (and) 2, four states $f_0(400-1200)$, $f_0(980)$, $f_0(1370)$ and $f_0(1500)$ are listed by Particle Data Group (PDG) [6]. There are too many controversies about these states, especially relating to the $f_0(400-1200)$ and $f_0(980)$. The convenient but not convincing recent tendency is to put aside the $f_0(400-1200)$ and $f_0(980)$, and focus on $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ [7], although the spin-parity of the $f_J(1710)$ $J^{PC}=0^{++}$ or (and) 2^{++} is controversial.

Recently, several authors have discussed the quarkonia-glueball content of the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ by studying the mixing of the three states [8–11]. The different assumption about the mass level order of the bare states $|G\rangle = |gg\rangle$, $|S\rangle = |s\bar{s}\rangle$ and $|N\rangle = |u\bar{u}+d\bar{d}\rangle/\sqrt{2}$ leads to the different quantitative predictions about the glueball-quarkonia content of the three states. In Ref. [9–11] $M_G > M_S > M_N$ is assumed and in Ref. [8] $M_S > M_G > M_N$ is assumed. The assumption, $M_G > M_{q\bar{q}}$, is consistent with the prediction given by lattice QCD [12] that the bare glueball state has a higher mass than the bare quarkonia states. However, without the confirmation that which state, $a_0(980)$ or $a_0(1450)$, is the isovector member in the ground $^3P_0$ nonet, the level order of the M_S and M_N in the scalar sector perhaps still remains open. On one hand, if the $a_0(1450)$ is assigned as the isovector member in the ground $^3P_0$ nonet, since the $a_0(1450)$ with mass 1474 ± 19 MeV [6] has a higher mass than the observed isodoublet scalar states $K_0^*(1430)$ with mass 1429 ± 6 MeV [6], according to the Gell-Mann- Okubo mass formula [13] one would expect $M_N > M_S$. On the other hand, if the $a_0(980)$ is assigned as the isovector member, one would expect $M_S > M_N$, which is also consistent with that the strange quark s has a higher mass than the non-strange quark u or d in a constituent quark picture. We believe that neither $M_S > M_N$ nor $M_N > M_S$ seems to be ruled out in the scalar sector in the current situation.

In this letter, based on the mixing scheme of the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$, we shall discuss the glueball-quarkonia content of the three states taking into account the two possible assumptions $M_G > M_S > M_N$ and $M_G > M_N > M_S$[1]. This paper is organized

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[1] Assuming $M_S = M_N$ and taking $M_1 = 1.712$ GeV, $M_2 = 1.5$ GeV, when M_3 changes from 1.2 to 1.5 GeV, from Eqs. (2)$\sim$(4), we find that $M_G > 0$ requires $M_S = M_N = 1.5$ GeV, which leads to $x_2 = y_2 = z_2 = 0$ in Eqs. (5), (6). Therefore, the possibility of $M_S = M_N$ can be ruled out.

as follows: In Sect. II, the two-body hadronic decays of $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$ are investigated in the quarkonia-glueball mixing framework. The results for two cases $M_G > M_S > M_N$ and $M_G > M_N > M_S$ are presented in Sect. III. Our conclusions are reached in Sect. IV.

II. MIXING SCHEME OF QUARKONIA AND GLUEBALL

In the $|G\rangle = |gg\rangle$, $|S\rangle = |s\bar{s}\rangle$, $|N\rangle = |u\bar{u} + d\bar{d}\rangle/\sqrt{2}$ basis, the mass matrix describing the mixing of a glueball and quarkonia can be written as follows [9]:

$$M = \begin{pmatrix} M_G & f & \sqrt{2}f \\ f & M_S & 0 \\ \sqrt{2}f & 0 & M_N \end{pmatrix}, \tag{1}$$

where $f = \langle G|M|S\rangle = \langle G|M|N\rangle/\sqrt{2}$ represents the flavor independent mixing strength between the glueball and quarkonia states. The vanishing off-diagonal elements indicate that there is no direct quarkonia mixing which is assumed to be a higher order effect. M_G, M_S and M_N represent the masses of the bare states $|G\rangle$, $|S\rangle$ and $|N\rangle$, respectively. Here we assume that the physical states $|f_0(1710)\rangle$, $|f_0(1500)\rangle$ and $|f_0(1370)\rangle$ are the eigenstates of M with the eigenvalues of M_1, M_2 and M_3, respectively (M_1, M_2 and M_3 denote the masses of $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$, respectively). If one defines a 3×3 unitary matrix U which transforms the states $|G\rangle$, $|S\rangle$ and $|N\rangle$ into the physical states $|f_0(1710)\rangle$, $|f_0(1500)\rangle$ and $|f_0(1370)\rangle$, then UMU^{-1} must be the diagonal matrix with the diagonal elements M_1, M_2 and M_3, from which one can get the following equations:

$$M_1 + M_2 + M_3 = M_G + M_S + M_N, \tag{2}$$

$$M_1M_2 + M_1M_3 + M_2M_3 = M_GM_S + M_GM_N + M_NM_S - 3f^2, \tag{3}$$

$$M_1M_2M_3 = M_GM_SM_N - f^2(2M_S + M_N). \tag{4}$$

The three physical states can be read as

$$\begin{pmatrix} |f_0(1710)\rangle \\ |f_0(1500)\rangle \\ |f_0(1370)\rangle \end{pmatrix} = U \begin{pmatrix} |G\rangle \\ |S\rangle \\ |N\rangle \end{pmatrix} = \begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix} \begin{pmatrix} |G\rangle \\ |S\rangle \\ |N\rangle \end{pmatrix}, \tag{5}$$

where

$$U = \begin{pmatrix} (M_1 - M_S)(M_1 - M_N)C_1 & (M_1 - M_N)fC_1 & \sqrt{2}(M_1 - M_S)fC_1 \\ (M_2 - M_S)(M_2 - M_N)C_2 & (M_2 - M_N)fC_2 & \sqrt{2}(M_2 - M_S)fC_2 \\ (M_3 - M_S)(M_3 - M_N)C_3 & (M_3 - M_N)fC_3 & \sqrt{2}(M_3 - M_S)fC_3 \end{pmatrix} \tag{6}$$

with $C_{i(i=1,\,2,\,3)} = [(M_i - M_S)^2(M_i - M_N)^2 + (M_i - M_N)^2 f^2 + 2(M_i - M_S)^2 f^2]^{-\frac{1}{2}}$.

In the above mixing scheme, for the hadronic decays of the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$, neglecting the possible glueball component in the final states, we consider the following coupling modes as shown in Fig. 1: the coupling of the quarkonia components of

the three states to the quarkonia components of the final state pseudoscalar mesons, and the coupling of the glueball components of the three states to the quarkonia components of the final state pseudoscalar mesons. Performing an elementary $SU(3)$ calculation [4,14–18], one can get the following equations:

$$\frac{\Gamma(f_0(1500)\to\eta\eta')}{\Gamma(f_0(1500)\to\eta\eta)}=\frac{P_{\eta\eta'}}{2P_{\eta\eta}}\frac{[4\alpha\beta(\frac{z_2}{\sqrt{2}}-y_2)]^2}{[(\sqrt{2}\alpha^2z_2+2\beta^2y_2)^2+2r\cos\theta(\sqrt{2}\alpha^2z_2+2\beta^2y_2)x_2+r^2x_2^2)]}, \quad (7)$$

$$\frac{\Gamma(f_0(1500)\to\pi^0\pi^0)}{\Gamma(f_0(1500)\to\eta\eta)}=\frac{P_{\pi^0\pi^0}}{P_{\eta\eta}}\frac{[\frac{z_2^2}{2}+\sqrt{2}r\cos\theta z_2x_2+r^2x_2^2]}{[(\sqrt{2}\alpha^2z_2+2\beta^2y_2)^2+2r\cos\theta(\sqrt{2}\alpha^2z_2+2\beta^2y_2)x_2+r^2x_2^2]}, \quad (8)$$

$$\frac{\Gamma(f_0(1500)\to K\bar{K})}{\Gamma(f_0(1500)\to\pi\pi)}=\frac{P_{KK}}{3P_{\pi\pi}}\frac{[(\frac{z_2}{\sqrt{2}}+y_2)^2+4r\cos\theta(\frac{z_2}{\sqrt{2}}+y_2)x_2+4r^2x_2^2]}{[\frac{z_2^2}{2}+\sqrt{2}r\cos\theta z_2x_2+r^2x_2^2]}, \quad (9)$$

$$\frac{\Gamma(f_0(1710)\to\pi\pi)}{\Gamma(f_0(1710)\to K\bar{K})}=3\frac{P'_{\pi\pi}}{P'_{KK}}\frac{[\frac{z_1^2}{2}+\sqrt{2}r\cos\theta z_1x_1+r^2x_1^2]}{[(\frac{z_1}{\sqrt{2}}+y_1)^2+4r\cos\theta(\frac{z_1}{\sqrt{2}}+y_1)+4r^2x_1^2]}, \quad (10)$$

$$\frac{\Gamma(f_0(1710)\to\eta\eta)}{\Gamma(f_0(1710)\to K\bar{K})}=\frac{P'_{\eta\eta}}{P'_{KK}}\frac{[(\sqrt{2}\alpha^2z_1+2\beta^2y_1)^2+2r\cos\theta(\sqrt{2}\alpha^2z_1+2\beta^2y_1)x_1+r^2x_1]}{[(\frac{z_1}{\sqrt{2}}+y_1)^2+4r\cos\theta(\frac{z_1}{\sqrt{2}}+y_1)x_1+4r^2x_1^2]}, \quad (11)$$

$$\frac{\Gamma(f_0(1710)\to\eta\eta')}{\Gamma(f_0(1710)\to K\bar{K})}=\frac{P'_{\eta\eta'}}{2P'_{KK}}\frac{[4\alpha\beta(\frac{z_1}{\sqrt{2}}-y_1)]^2}{[(\frac{z_1}{\sqrt{2}}+y_1)^2+4r\cos\theta(\frac{z_1}{\sqrt{2}}+y_1)x_1+4r^2x_1^2]}, \quad (12)$$

$$\frac{\Gamma(f_0(1370)\to\pi\pi)}{\Gamma(f_0(1370)\to K\bar{K})}=3\frac{P''_{\pi\pi}}{P''_{KK}}\frac{[\frac{z_3^2}{2}+\sqrt{2}r\cos\theta z_3x_3+r^2x_3^2]}{[(\frac{z_3}{\sqrt{2}}+y_3)^2+4r\cos\theta(\frac{z_3}{\sqrt{2}}+y_3)x_3+4r^2x_3^2]}, \quad (13)$$

$$\frac{\Gamma(f_0(1370)\to\eta\eta)}{\Gamma(f_0(1370)\to K\bar{K})}=\frac{P''_{\eta\eta}}{P''_{KK}}\frac{[(\sqrt{2}\alpha^2z_3+2\beta^2y_3)^2+2r\cos\theta(\sqrt{2}\alpha^2z_3+2\beta^2y_3)x_3+r^2x_3^2]}{[(\frac{z_3}{\sqrt{2}}+y_3)^2+4r\cos\theta(\frac{z_3}{\sqrt{2}}+y_3)x_3+4r^2x_3^2]}, \quad (14)$$

where $\alpha=(\cos\theta_p-\sqrt{2}\sin\theta_p)/\sqrt{6}$, $\beta=(\sin\theta_p+\sqrt{2}\cos\theta_p)/\sqrt{6}$, θ_p is the mixing angle of isoscalar octet-singlet for pseudoscalar mesons; $P_{jj'}$ ($P'_{jj'}$, $P''_{jj'}$) ($j,j'=\pi,\ \eta,\ \eta',\ K$) is the momentum of the final state meson in the center of mass system for the jj' decays of the $f_0(1500)$ ($f_0(1710)$, $f_0(1370)$); r represents the ratio of the effective coupling strength of the coupling mode (b) to that of the coupling mode (a); θ is the relative phase between the amplitude of the coupling mode (b) and that of the coupling mode (a).

For the two-photon decays of the three states, one can get [19]

$$\begin{aligned}&\Gamma(f_0(1710)\to\gamma\gamma):\Gamma(f_0(1500)\to\gamma\gamma):\Gamma(f_0(1370)\to\gamma\gamma)=\\&M_1^3(5z_1+\sqrt{2}y_1)^2:M_2^3(5z_2+\sqrt{2}y_2)^2:M_3^3(5z_3+\sqrt{2}y_3)^2\end{aligned} \quad (15)$$

III. THE RESULTS FOR THE CASES $M_G>M_S>M_N$ AND $M_G>M_N>M_S$

The decay data relating to the $f_0(1500)$ are as follows [20,21]:

$$\begin{aligned}&\Gamma(f_0(1500)\to\eta\eta')/\Gamma(f_0(1500)\to\eta\eta)=0.84\pm0.23,\\&\Gamma(f_0(1500)\to\pi^0\pi^0)/\Gamma(f_0(1500)\to\eta\eta)=4.29\pm0.72,\\&\Gamma(f_0(1500)\to K\bar{K})/\Gamma(f_0(1500)\to\pi\pi)=0.19\pm0.07.\end{aligned} \quad (16)$$

The decay datum of the $f_0(1710)$ is [6]

$$\Gamma(\pi\pi)/\Gamma(K\bar{K}) = 0.39 \pm 0.14. \tag{17}$$

Apart from $M_1 = 1.712$ GeV and $M_2 = 1.5$ GeV, the central values of the masses of the $f_0(1710)$ and $f_0(1500)$, respectively [6], we take the central values of the decay data mentioned above and $\theta_p = -19.1^\circ$ [22,23] as input. In this way seven parameters, M_G, M_3, M_N, M_S, f, θ and r are unknown. We perform numerically to solve the unlinear equations (2) $\sim$ (4), (7) $\sim$ (10) for the cases $M_G > M_S > M_N$ and $M_G > M_N > M_S$, respectively. The two solutions are presented in Table I.

Table I shows that in both cases, the mass of the pure glueball is 1.590 GeV, which is in agreement with the lattice QCD simulations which give 1.55 ± 0.05 GeV [24] and 1.63 ± 0.08 GeV [12,25] for the scalar glueball mass. In addition, from Table I the masses of the pure $|N\rangle$ and $|S\rangle$ are close to the mass of the pure glueball in both cases, which implies that a large mixing would exist on $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. We note that the mass of $f_0(1370)$ is determined to be the value of 1.200 GeV in the case $M_G > M_S > M_N$ and 1.380 GeV in the case $M_G > M_N > M_S$, which is also consistent with 1.200 $\sim$ 1.500 GeV estimated by PDG [6].

For the case $M_G > M_S > M_N$, the numerical form of the unitary matrix U is

$$U = \begin{pmatrix} 0.793 & 0.550 & 0.263 \\ -0.473 & 0.830 & -0.296 \\ 0.382 & -0.112 & -0.917 \end{pmatrix}. \tag{18}$$

The physical states $|f_0(1710)\rangle$, $|f_0(1500)\rangle$ and $|f_0(1370)\rangle$ can be read as

$$|f_0(1710)\rangle = 0.793|G\rangle + 0.550|S\rangle + 0.263|N\rangle, \tag{19}$$
$$|f_0(1500)\rangle = -0.473|G\rangle + 0.830|S\rangle - 0.296|N\rangle, \tag{20}$$
$$|f_0(1370)\rangle = 0.382|G\rangle - 0.112|S\rangle - 0.917|N\rangle, \tag{21}$$

which indicates that in the case $M_G > M_S > M_N$, $f_0(1710)$ ($f_0(1500)$, $f_0(1370)$) contains about 63% (22%, 15%) glueball component, 30% (69%, 1%) $s\bar{s}$ component and 7% (9%, 84%) $(u\bar{u}+d\bar{d})/\sqrt{2}$ component. In the case $M_G > M_S > M_N$ our results are consistent with the results given by Ref. [9–11].

Based on Eqs. (7)$\sim$(15) as well as Eqs. (19)$\sim$(21), the numerical results relating to the hadronic decays of the $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$ in the case $M_G > M_S > M_N$ are shown in the Table II, and the two-photon decay width ratio for the three states is given by

$$\Gamma_{\gamma\gamma}(f_0(1710)) : \Gamma_{\gamma\gamma}(f_0(1500)) : \Gamma_{\gamma\gamma}(f_0(1370)) = 21.917 : 0.316 : 38.901. \tag{22}$$

For the case $M_G > M_N > M_S$, the numerical form of the unitary matrix U is

$$U = \begin{pmatrix} 0.748 & 0.220 & 0.626 \\ -0.445 & -0.527 & 0.724 \\ 0.493 & -0.816 & -0.301 \end{pmatrix}. \tag{23}$$

The physical states $|f_0(1710)\rangle$, $|f_0(1500)\rangle$ and $|f_0(1370)\rangle$ can be read as

$$|f_0(1710)\rangle = 0.748|G\rangle + 0.220|S\rangle + 0.626|N\rangle, \tag{24}$$
$$|f_0(1500)\rangle = -0.445|G\rangle - 0.527|S\rangle + 0.724|N\rangle, \tag{25}$$
$$|f_0(1370)\rangle = 0.493|G\rangle - 0.816|S\rangle - 0.301|N\rangle. \tag{26}$$

which indicates in the case $M_G > M_N > M_S$, $f_0(1710)$ ($f_0(1500)$, $f_0(1370)$) contains about 56% (20%, 24%) glueball component, 5% (28%, 67%) $s\bar{s}$ component and 39% (52%, 9%) $(u\bar{u} + d\bar{d})/\sqrt{2}$ component.

Similarly, based on Eqs. (7)∼(15) as well as Eqs. (24)∼(26), the numerical results relating to the hadronic decays of the $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$ are shown in the Table III, and the two-photon decay width ratio for the three states is given by

$$\Gamma_{\gamma\gamma}(f_0(1710)) : \Gamma_{\gamma\gamma}(f_0(1500)) : \Gamma_{\gamma\gamma}(f_0(1370)) = 59.388 : 27.908 : 18.564. \tag{27}$$

From Eqs. (18) and (23), in both cases, a large mixing effect on the $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$ exists, which is consistent with our above results that in the scalar sector, pure glueball and quarkonia lie in a vicinal mass region. The largest components of $f_0(1370)$ and $f_0(1500)$ are quarkonia and the quarkonia content of the $f_0(1500)$ and $f_0(1370)$ differs due to the different mass level order of the bar states $|N\rangle$ and $|S\rangle$, which is consistent with the main property of the mass matrix (1) that upon mixing the higher mass bare state becomes more massive, while the lower mass bare state becomes less massive (i.e., the mass splitting between the higher and lower mass bare states increases as a result of the mixing) [11]. The largest component of $f_0(1710)$ is glueball, which supports the argument that $f_J(1710)$ is a mixed $q\bar{q}$ glueball having a large glueball component if its spin is determined to be 0 [8]. Furthermore, in both cases, the results exhibit destructive interference between the states $|N\rangle$ and $|S\rangle$ for $f_0(1500)$ while constructive interference between the states $|N\rangle$ and $|S\rangle$ for $f_0(1710)$ and $f_0(1370)$, which is also consistent with the conclusion given by Ref. [8–11].

The predictions about the decays of the three states in two cases can provide a stringent consistency check of the two assumptions, therefore measurements of the above decay channels of the three states, especially the $\eta\eta'$ decay channel of $f_0(1710)$ and the $\pi\pi$ decay channel of $f_0(1370)$ as well as the two-photon decays of the three states, can be most relevant in clarifying which assumption is really reasonable. In addition, it is important to investigate the nature of the $a_0(1450)$ and $a_0(980)$, since the isovector scalar state and the isodoublet scalar state K_0^* can set a natural mass scale of the ground scalar meson nonet.

IV. SUMMARY AND CONCLUSIONS

In the scalar glueball-quarkonia mixing framework, we study the two-hadronic decays of $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ considering the coupling quarkonia and glueball components of the three states to the quarkonia components of the final states pseudoscalar mesons. Taking into account two possible assumptions about the mass level order of the bare states $|G\rangle$, $|S\rangle$ and $|N\rangle$, $M_G > M_S > M_N$ and $M_G > M_N > M_S$, we obtain the quarkonia-glueball content of the three states by solving the unlinear equations.

Our conclusions are as follows:

1). In the scalar sector, the pure glueball and quarkonia have comparable masses and a significant mixing of glueball with the isoscalar mesons exists.

2). The largest component of $f_0(1710)$ is glueball (about 60%) and the largest component of $f_0(1500)$ ($f_0(1370)$) is quarkonia. Which flavor $(u\bar{u}+d\bar{d})/\sqrt{2}$ or $s\bar{s}$ is dominant component of $f_0(1500)$ ($f_0(1370)$) depends on the mass level order of the pure states $|N\rangle$ and $|S\rangle$.

3). The interference between $|N\rangle$ and $|S\rangle$ is destructive for $f_0(1500)$ while constructive for $f_0(1370)$ and $f_0(1710)$.

4). The measurements for the $\eta\eta'$ decay channel of $f_0(1710)$ and the $\pi\pi$ decay channel of $f_0(1370)$ as well as the two-photon decays of the three states would be relevant to judge which assumption is really reasonable. Moreover, the confirmation of the nature about the $a_0(980)$ and $a_0(1450)$ would be useful to check the consistency of two assumptions.

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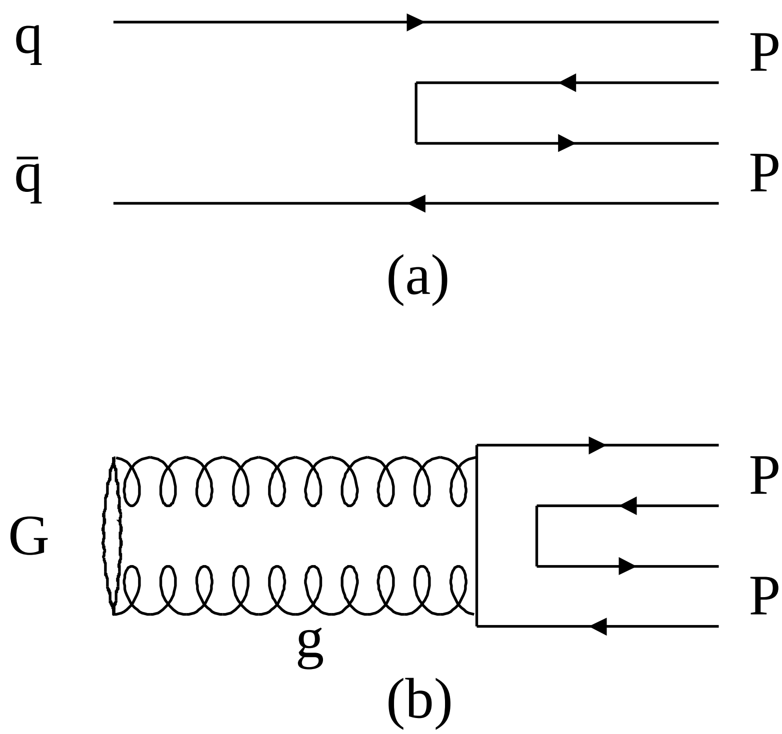

FIG. 1. (a): The coupling of the quarkonia component $q\bar{q}$ in the decaying particles to the final state pseudoscalar mesons PP. (b): The coupling of the glueball component G in the decaying particles to the final state pseudoscalar mesons PP.

TABLES

Parameters	$M_G > M_S > M_N$	$M_G > M_N > M_S$
M_G (GeV)	1.590	1.590
M_S (GeV)	1.560	1.430
M_N (GeV)	1.262	1.572
M_3 (GeV)	1.200	1.380
f (GeV)	0.105	0.083
$\cos\theta$	0.906	-0.974
r	1.220	0.740

TABLE I. The solutions to the Eqs. (2) $\sim$ (4), (7) $\sim$ (10) for the cases $M_G > M_S > M_N$ and $M_G > M_N > M_S$.

$f_0(1710)$			$f_0(1500)$			$f_0(1370)$		
Modes	Exp.	Theor.	Modes	Exp.	Theor.	Modes	Exp.	Theor.
$\frac{\Gamma(\pi\pi)}{\Gamma(K\bar{K})}$	0.39 ± 0.14	0.687	$\frac{\Gamma(\eta\eta')}{\Gamma(\eta\eta)}$	0.84 ± 0.23	0.905	$\frac{\Gamma(\pi\pi)}{\Gamma(K\bar{K})}$		2.858
$\frac{\Gamma(\eta\eta)}{\Gamma(K\bar{K})}$		0.216	$\frac{\Gamma(\pi^0\pi^0)}{\Gamma(\eta\eta)}$	4.29 ± 0.72	4.225	$\frac{\Gamma(\eta\eta)}{\Gamma(K\bar{K})}$		0.181
$\frac{\Gamma(\eta\eta')}{\Gamma(K\bar{K})}$		0.005	$\frac{\Gamma(K\bar{K})}{\Gamma(\pi\pi)}$	0.19 ± 0.07	0.178			

TABLE II. The numerical results as well as the experimental data relating to the decays of the $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$ for the case $M_G > M_S > M_N$.

$f_0(1710)$			$f_0(1500)$			$f_0(1370)$		
Modes	Exp.	Theor.	Modes	Exp.	Theor.	Modes	Exp.	Theor.
$\frac{\Gamma(\pi\pi)}{\Gamma(K\bar{K})}$	0.39 ± 0.14	0.385	$\frac{\Gamma(\eta\eta')}{\Gamma(\eta\eta)}$	0.84 ± 0.23	0.778	$\frac{\Gamma(\pi\pi)}{\Gamma(K\bar{K})}$		0.454
$\frac{\Gamma(\eta\eta)}{\Gamma(K\bar{K})}$		0.181	$\frac{\Gamma(\pi^0\pi^0)}{\Gamma(\eta\eta)}$	4.29 ± 0.72	4.269	$\frac{\Gamma(\eta\eta)}{\Gamma(K\bar{K})}$		0.173
$\frac{\Gamma(\eta\eta')}{\Gamma(K\bar{K})}$		0.054	$\frac{\Gamma(K\bar{K})}{\Gamma(\pi\pi)}$	0.19 ± 0.07	0.151			

TABLE III. The numerical results as well as the experimental data relating to the decays of the $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$ for the case $M_G > M_N > M_S$.